

A simple model for high-energy nucleon-nucleon elastic diffraction and exclusive diffractive electroproduction of vector mesons on protons

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Abstract

The processes of exclusive diffractive scattering $p + p \rightarrow p + p$, $\bar{p} + p \rightarrow \bar{p} + p$, and $\gamma^* + p \rightarrow V + p$ at high energies are considered in the framework of a unified Regge-eikonal model with a very simple reggeon structure of the eikonal. It is demonstrated that the pomeron trajectory is universal in all reactions and having intercept about 1.31 which could be extracted explicitly from the data on the proton structure function $F_2(x, Q^2)$. The predictions for the proton-proton cross-sections at LHC energies are given.

Introduction

The aim of this job is the construction of quite a simple phenomenological model which could be applicable to various processes of high energy diffractive scattering. For this purpose we will consider reactions of nucleon-nucleon elastic diffraction and exclusive electroproduction of vector mesons on protons and try to give a simultaneous description of these processes. This will be done in the framework of the Regge-eikonal approach [1] which allows to satisfy the Froissart-Martin bound [2] explicitly. The basis of this approach is the eikonal (impact parameter) representation of the elastic diffractive non-flip scattering amplitude

$$T_{12 \rightarrow 12}(s, t) = 4\pi\lambda^{1/2}(s, m_1^2, m_2^2) \int_0^\infty db^2 J_0(b\sqrt{-t}) \frac{e^{2i\delta_{12 \rightarrow 12}(s, b)} - 1}{2i}, \quad (1)$$

where s is the collision energy squared, t is the transferred 4-momentum squared, b is the impact parameter, m_1 and m_2 are the masses of the scattering particles, $\lambda(s, m_1^2, m_2^2) \equiv s^2 + m_1^4 + m_2^4 - 2m_1^2s - 2m_2^2s - 2m_1^2m_2^2$, and the eikonal is the sum of single-reggeon exchange terms [1] (for derivation of the eikonal Regge approximation see, also, Appendix):

$$\begin{aligned} \delta_{12 \rightarrow 12}(s, b) &= \frac{1}{16\pi\lambda^{1/2}(s, m_1^2, m_2^2)} \int_0^\infty d(-t) J_0(b\sqrt{-t}) \delta_{12 \rightarrow 12}(s, t) = \frac{1}{16\pi\lambda^{1/2}(s, m_1^2, m_2^2)} \times \\ &\times \int_0^\infty d(-t) J_0(b\sqrt{-t}) \left\{ \sum_n \left(i + \operatorname{tg} \frac{\pi(\alpha_n^+(t) - 1)}{2} \right) \Gamma_n^{(1)+}(t) \Gamma_n^{(2)+}(t) \left(\frac{s}{s_0} \right)^{\alpha_n^+(t)} \mp \right. \end{aligned} \quad (2)$$

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$$\mp \sum_n \left(i - \text{ctg} \frac{\pi(\alpha_n^-(t) - 1)}{2} \right) \Gamma_n^{(1)-}(t) \Gamma_n^{(2)-}(t) \left(\frac{s}{s_0} \right)^{\alpha_n^-(t)} \Bigg\}.$$

Here $\alpha_n^+(t)$, $\Gamma_n^{(i)+}(t)$ and $\alpha_n^-(t)$, $\Gamma_n^{(i)-}(t)$ are C -even and C -odd Regge trajectories and reggeon form-factors of the scattered particles, $s_0 \equiv 1 \text{ GeV}^2$, and the sign “ $-$ ” (“ $+$ ”) before C -odd contributions corresponds to the particle-particle (particle-antiparticle) scattering.

The practical use of the Regge-eikonal approach is that in the case of high energy diffraction it allows to reduce the unknown function of two variables, $T_{12 \rightarrow 12}(s, t)$, to a few functions of one dynamical variable t (Regge trajectories and reggeon form-factors) and to make explicit estimations for the high energy evolution of the diffractive pattern. In the following sections we will demonstrate that for various diffractive reactions there exist wide kinematical ranges where for description of cross-sections it is enough to keep only two C -even reggeons in the eikonal, pomeron and f_2 -reggeon.

Since the behavior of Regge trajectories and reggeon form-factors at low values of t is still not calculable in the framework of QCD we have to use for them purely phenomenological (test) expressions. But though the main criterion for our choice of such parametrizations is simplicity we should take into account the QCD asymptotic behavior of Regge trajectories at $t \rightarrow -\infty$. For example, if we assume that in the limit of large transfers the pomeron exchanges turn into multi-gluon exchanges then (like in QED [3]) we come to [4, 5, 6]

$$\lim_{t \rightarrow -\infty} \alpha_P(t) = 1. \quad (3)$$

In the case of the quark-antiquark pair (f_2 -reggeon, *etc.*) one obtains [7]

$$\lim_{t \rightarrow -\infty} \alpha_{\bar{q}q}(t) \ln^{1/2}(-t) = \sqrt{\frac{32}{11N_c - 2n_f}} = \sqrt{\frac{32}{21}}, \quad (4)$$

where $N_c = 3$ is the number of colors and $n_f = 6$ is the full number of quark flavors.

The main feature of the proposed model is the use of universal (“universality” means “uniqueness”, *i.e.* that corresponding reggeons are the same in applications of the model to various reactions) and essentially nonlinear Regge trajectories with asymptotical behavior satisfying (3), (4).

The model for the nucleon-nucleon elastic diffraction

Thus, for the nucleon-nucleon elastic diffraction we will exploit the following approximation to the eikonal in the momentum representation:

$$\begin{aligned} \delta(s, t) = \delta_P(s, t) + \delta_f(s, t) = & \left(i + \text{tg} \frac{\pi(\alpha_P(t) - 1)}{2} \right) \Gamma_P^{(\text{pp})^2}(t) \left(\frac{s}{s_0} \right)^{\alpha_P(t)} + \\ & + \left(i + \text{tg} \frac{\pi(\alpha_f(t) - 1)}{2} \right) \Gamma_f^{(\text{pp})^2}(t) \left(\frac{s}{s_0} \right)^{\alpha_f(t)}. \end{aligned} \quad (5)$$

For the pomeron and the f_2 -reggeon trajectories we choose the simplest parametrizations¹ satisfying asymptotical conditions (3), (4):

$$\alpha_P(t) = 1 + C_P e^{B_P t}, \quad \alpha_f(t) = \sqrt{\frac{32}{21}} \ln^{-1/2} \frac{t_f - t}{\Lambda_f^2}. \quad (6)$$

¹One should not consider analytical properties of test parametrizations seriously. True Regge trajectories and reggeon form-factors have much more complicated analytical structure. Nevertheless, at negative values of the argument they could be approximated by simple monotonic test functions.

The reggeon test form-factors are chosen as

$$\Gamma_P^{(pp)}(t) = \Gamma_P^{(pp)} e^{b_P^{(pp)} t}, \quad \Gamma_f^{(pp)}(t) = \Gamma_f^{(pp)} e^{b_f^{(pp)} t}. \quad (7)$$

To obtain angular distributions one should substitute (6), (7) into (5), then using (2) and (1) calculate the scattering amplitude (integrals are calculated numerically) and substitute it into the expression for the differential cross-section

$$\frac{d\sigma_{el}}{dt} = \frac{|T_{el}(s, t)|^2}{16\pi\lambda(s, m_p^2, m_p^2)}. \quad (8)$$

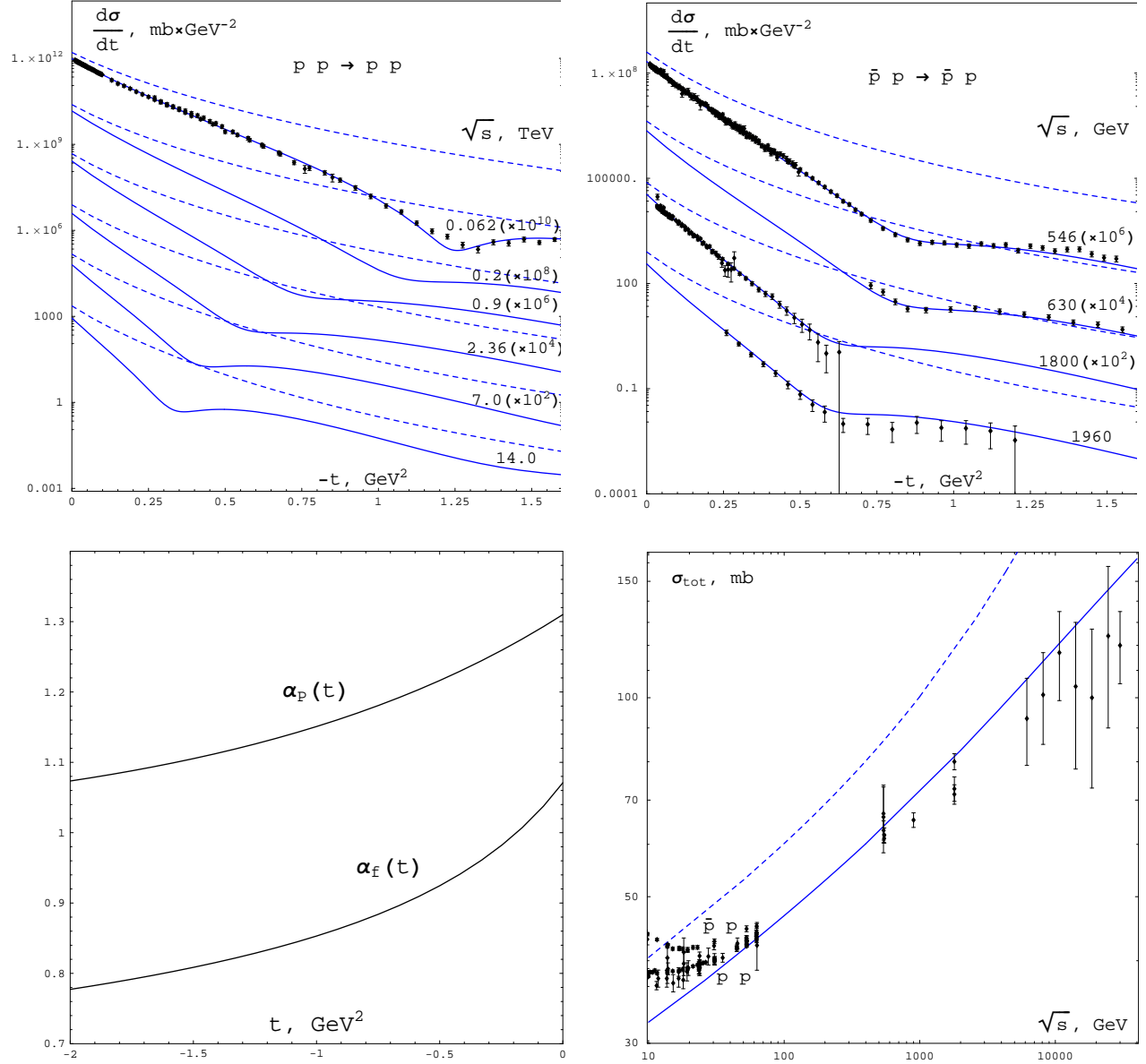


Figure 1: Nucleon-nucleon cross-sections at different values of the collision energy and approximate Regge trajectories of the pomeron and the f_2 -reggeon. The dashed lines correspond to the Born amplitudes.

We postpone the discussion of the used parametrizations to the last section and now apply the proposed phenomenological scheme to the description of the elastic nucleon-nucleon diffractive scattering. Here we restrict ourselves by the kinematical range $\sqrt{s} > 60$ GeV, 0.01

$\text{GeV}^2 < -t < 1.6 \text{ GeV}^2$. At larger transfers the systematic deviations of the model curves from experimental angular distributions become too large due to the fact that exponential approximations to reggeon form-factors are invalid in the hard scattering region. At $\sqrt{s} < 60 \text{ GeV}$ the contributions from secondary reggeons (ω , ρ , a , *etc.*) have so significant influence on the values of dynamical quantities at low t and in the region of the first dip that they also should not be ignored.² And, at last, in the region $\sqrt{-t} < 0.1 \text{ GeV}$ the interference with Coulomb interaction takes place.

The value of the pomeron intercept can be extracted explicitly from the data on γ^*p total cross-sections at high photon virtualities and turns out ≈ 1.31 . We will discuss this below in one of the following sections.

For verification of the model we used the set of data on angular distributions collected by J.R. Cudell, A. Lengyel, and E. Martynov at [8]. The original data can also be found at [9]. The data set for $\sqrt{s} = 1.96 \text{ TeV}$ is taken from [10]. The results of applying the model to the data are represented in Fig. 1 and Tabs. 1, 2.

pomeron		f -reggeon	
C_P	0.31 (FIXED)	t_f	0.87 GeV^2
B_P	0.72 GeV^{-2}	Λ_f	0.48 GeV
$\Gamma_P^{(pp)}$	0.95	$\Gamma_f^{(pp)}$	8.5
$b_P^{(pp)}$	0.24 GeV^{-2}	$b_f^{(pp)}$	1.15 GeV^{-2}

Table 1: Parameter values for test parametrizations of leading Regge trajectories and corresponding reggeon form-factors of nucleons.

Set of data	Number of points	χ^2
$\sqrt{s} = 62 \text{ GeV } (pp)$	122	112
$\sqrt{s} = 546 \text{ GeV } (\bar{p}p)$	221	291
$\sqrt{s} = 630 \text{ GeV } (\bar{p}p)$	13	13
$\sqrt{s} = 1800 \text{ GeV } (\bar{p}p, \text{E-710})$	51	68
$\sqrt{s} = 1800 \text{ GeV } (\bar{p}p, \text{CDF})$	26	77
$\sqrt{s} = 1960 \text{ GeV } (\bar{p}p, \text{D0})$	17	25
Total	450	586

Table 2: The quality of description of the data on angular distributions of elastic nucleon-nucleon scattering.

Also, in Fig. 1 the total cross-section dependence on the collision energy [11] is given. In particular, $\sigma_{tot}^{pp}(7 \text{ TeV}) \approx 110 \text{ mb}$ and $\sigma_{tot}^{pp}(14 \text{ TeV}) \approx 128 \text{ mb}$.

The model for the exclusive diffractive electroproduction of vector mesons on protons

We will treat exclusive diffractive electroproduction of vector mesons on protons from the standpoint of the vector-dominance model (VDM) [12] where the incoming photon fluctuates

²In fact, relative deviations about even one percent from the experimental data are noticeable in the near-forward scattering vicinity. So, we take into consideration only pp -scattering at $\sqrt{s} \approx 62 \text{ GeV}$ since at such energy the influence of secondary reggeons is minimal in comparison with other ISR data sets, also due to the fact that C -odd and C -even secondary contributions to the eikonal have opposite signs and compensate one another.

into a virtual vector meson which, in turn, scatters from the target proton. According to the so-called hypothesis of s -channel helicity conservation (the adequacy of this approximation is confirmed by numerous experimental data on helicity effects [13, 14]) the cross-section of reaction $\gamma^* + p \rightarrow V + p$ (here V denotes some vector meson) is dominated by the non-flip helicity amplitude which can be represented in the form

$$T_{\gamma^*p \rightarrow Vp}^\lambda(W^2, t, Q^2) = \sum_{V'} C_{V'}^\lambda(Q^2) T_{V'^*p \rightarrow Vp}^\lambda(W^2, t, Q^2), \quad (9)$$

where W is the collision energy, Q^2 is the incoming photon virtuality, $T_{V'^*p \rightarrow Vp}^\lambda(W^2, t, Q^2)$ is the diffractive hadronic amplitude with Regge-eikonal structure (the applicability of the Regge-eikonal approach to the hadronic reactions with off-shell particles is grounded in [15]), and the sum is taken over all neutral vector mesons. At low Q^2 the function $C_{V'}^\lambda(Q^2)$ behaves like the coefficient of vector dominance (at high Q^2 the VDM is invalid [13, 14]) dependent on the type of vector meson V' , helicity λ , and virtuality Q^2 .

It is known that for small enough values of t the diagonal coupling of vacuum reggeons to hadrons is much stronger than the off-diagonal coupling (for example, elastic proton-proton scattering has a considerably larger cross-section than diffractive excitation of $N(1470)$ in the proton-proton collisions). Hence, in the high-energy range ($W > 30$ GeV) where vacuum exchanges exceed the non-vacuum ones the so-called “diagonal approximation” of (9) may be used

$$T_{\gamma^*p \rightarrow Vp}^\lambda(W^2, t, Q^2) = C_V^\lambda(Q^2) T_{V^*p \rightarrow Vp}(W^2, t, Q^2). \quad (10)$$

Here we should note that measurements by ZEUS Collaboration [13, 14] did not reveal patent dependence on t and W of the ratio of differential cross-sections for the cases of longitudinally and transversely polarized incoming photon and this is the reason why we have omitted the superscript λ for $T_{V^*p \rightarrow Vp}(W^2, t, Q^2)$ in (10). In other words, we presume that spin phenomena related to the dependence of $T_{V^*p \rightarrow Vp}(W^2, t, Q^2)$ on helicities of incoming and outgoing particles are much more fine effects than diffractive scattering itself and further deal only with amplitude $T_{\gamma^*p \rightarrow Vp}(W^2, t, Q^2)$ averaged over spin states (it is obtained by the replacement $C_V^\lambda(Q^2) \rightarrow \sqrt{\frac{1}{3} \sum_\lambda [C_V^\lambda(Q^2)]^2}$ in (10)).³ For this quantity the extended (off-shell) eikonal representation is valid [15]:

$$\begin{aligned} T_{\gamma^*p \rightarrow Vp}(W^2, b, Q^2) &= \frac{\delta_{\gamma^*p \rightarrow Vp}(W^2, b, Q^2)}{\delta_{Vp \rightarrow Vp}(W^2, b)} T_{Vp \rightarrow Vp}(W^2, b) = \\ &= \delta_{\gamma^*p \rightarrow Vp}(W^2, b, Q^2) + i\delta_{\gamma^*p \rightarrow Vp}(W^2, b, Q^2)\delta_{Vp \rightarrow Vp}(W^2, b) + \dots \end{aligned} \quad (11)$$

(where b is the impact parameter, $\delta_{\gamma^*p \rightarrow Vp}(W^2, b, Q^2)$ is the “eikonal” (the sum of pole terms) of the vector meson electroproduction on protons, $\delta_{Vp \rightarrow Vp}(W^2, b)$ is the eikonal of the vector meson elastic scattering on protons, and $T_{Vp \rightarrow Vp}(W^2, b) = \frac{e^{2i\delta_{Vp \rightarrow Vp}(W^2, b)} - 1}{2i}$ is the “eikonalized” (unitarized) amplitude of elastic Vp -scattering).

The secondary reggeon exchanges are suppressed (C -odd reggeon exchanges – due to the C -parity conservation, a_2 -reggeon exchange – due to the isospin conservation). Hence, the pole part of the amplitude takes a relatively simple form:

$$\begin{aligned} \delta_{\gamma^*p \rightarrow Vp}(W^2, t, Q^2) &= \left[\left(i + \text{tg} \frac{\pi(\alpha_P(t) - 1)}{2} \right) \Gamma_P^{(V^*V)}(t, Q^2) \Gamma_P^{(pp)}(t) \left(\frac{W^2}{W_0^2} \right)^{\alpha_P(t)} + \right. \\ &\quad \left. + \left(i + \text{tg} \frac{\pi(\alpha_f(t) - 1)}{2} \right) \Gamma_f^{(V^*V)}(t, Q^2) \Gamma_f^{(pp)}(t) \left(\frac{W^2}{W_0^2} \right)^{\alpha_f(t)} \right] \sqrt{\frac{3\Gamma_{V \rightarrow e^+e^-}}{\alpha_e M_V}} \frac{M_V^2}{M_V^2 + Q^2}, \end{aligned} \quad (12)$$

³The Q^2 -behavior of $C_V^\lambda(Q^2)$ differs at different λ but in this job we concentrate on the reggeon structure of the amplitude and, so, on the helicity independent W - and t -behavior of cross-sections.

where M_V is the vector meson mass, $\alpha_e \approx \frac{1}{137}$ is the electromagnetic coupling, $\Gamma_{V \rightarrow e^+e^-}$ is the width of the vector meson decay to the electron-positron pair, $W_0 \equiv 1$ GeV, $\alpha_P(t)$ and $\alpha_f(t)$ are the pomeron and the f_2 -reggeon trajectories, $\Gamma_P^{(pp)}(t)$ and $\Gamma_f^{(pp)}(t)$ are the corresponding reggeon form-factors of the proton (the factor $\sqrt{\frac{3\Gamma_{V \rightarrow e^+e^-}}{\alpha_e M_V}} \frac{M_V^2}{M_V^2 + Q^2}$ is singled out explicitly from the photon-meson-reggeon vertex functions for convenience). The VDM implies that $\Gamma_R^{(V^*V)}(t, -M_V^2) =$

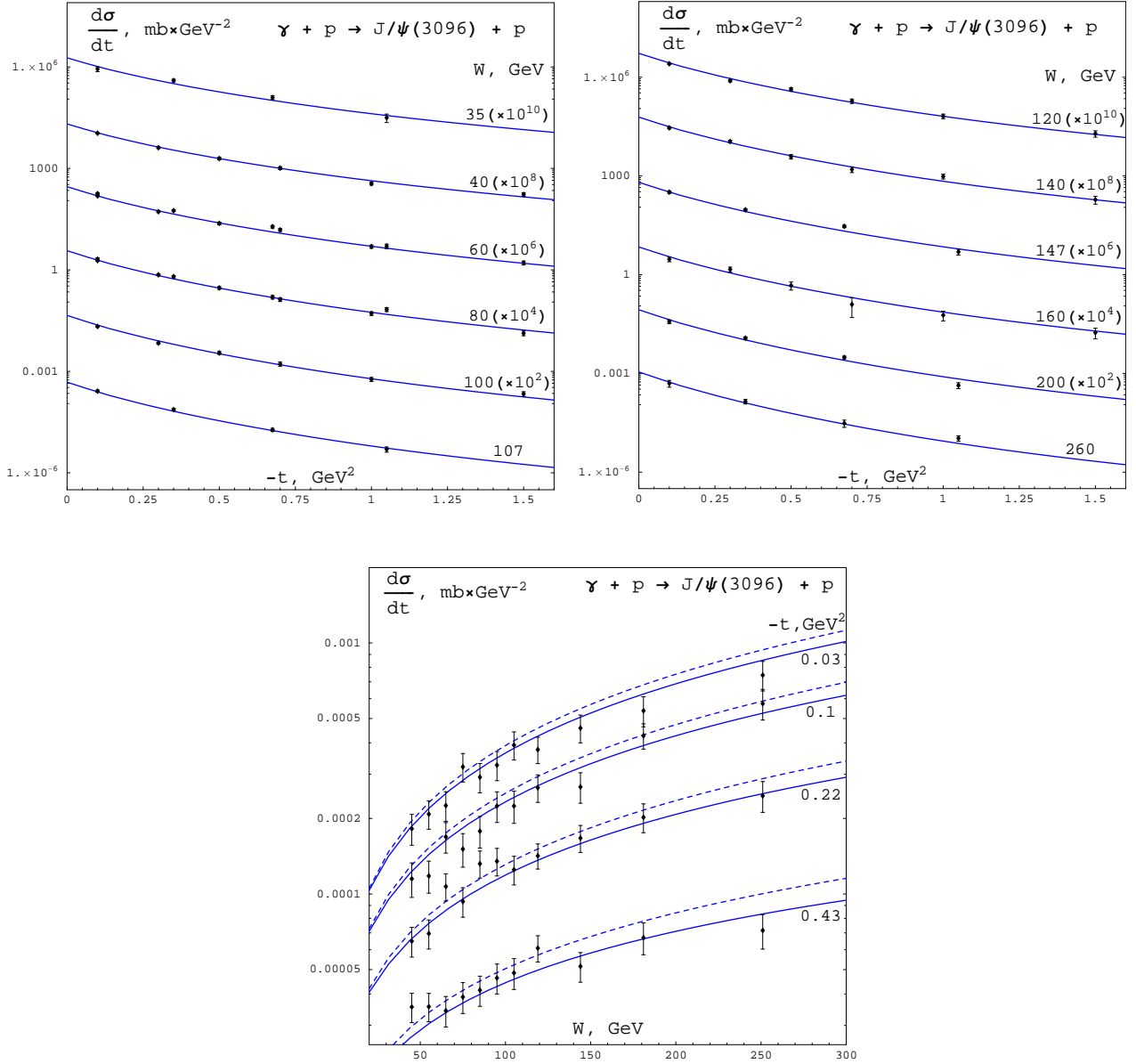


Figure 2: Differential cross-sections of the $J/\psi(3096)$ photoproduction at different values of the collision energy. The dashed lines correspond to the Born amplitudes.

$\Gamma_R^{(V^*V)}(t)$, where $\Gamma_R^{(V^*V)}(t)$ is the reggeon ($R = P, f$) form-factor of the vector meson:

$$\delta_{Vp \rightarrow Vp}(W^2, t) = \left(i + \text{tg} \frac{\pi(\alpha_P(t) - 1)}{2} \right) \Gamma_P^{(V^*V)}(t) \Gamma_P^{(pp)}(t) \left(\frac{W^2}{W_0^2} \right)^{\alpha_P(t)} \\ \left(i + \text{tg} \frac{\pi(\alpha_f(t) - 1)}{2} \right) \Gamma_f^{(V^*V)}(t) \Gamma_f^{(pp)}(t) \left(\frac{W^2}{W_0^2} \right)^{\alpha_f(t)}. \quad (13)$$

To calculate $T_{\gamma^*p \rightarrow Vp}(W^2, t, Q^2)$ we should determine functions $\Gamma_R^{(V^*V)}(t, Q^2)$ and $\Gamma_R^{(VV)}(t)$ (functions $\alpha_R(t)$ and $\Gamma_R^{(pp)}(t)$ were fixed under consideration of elastic nucleon-nucleon scattering).

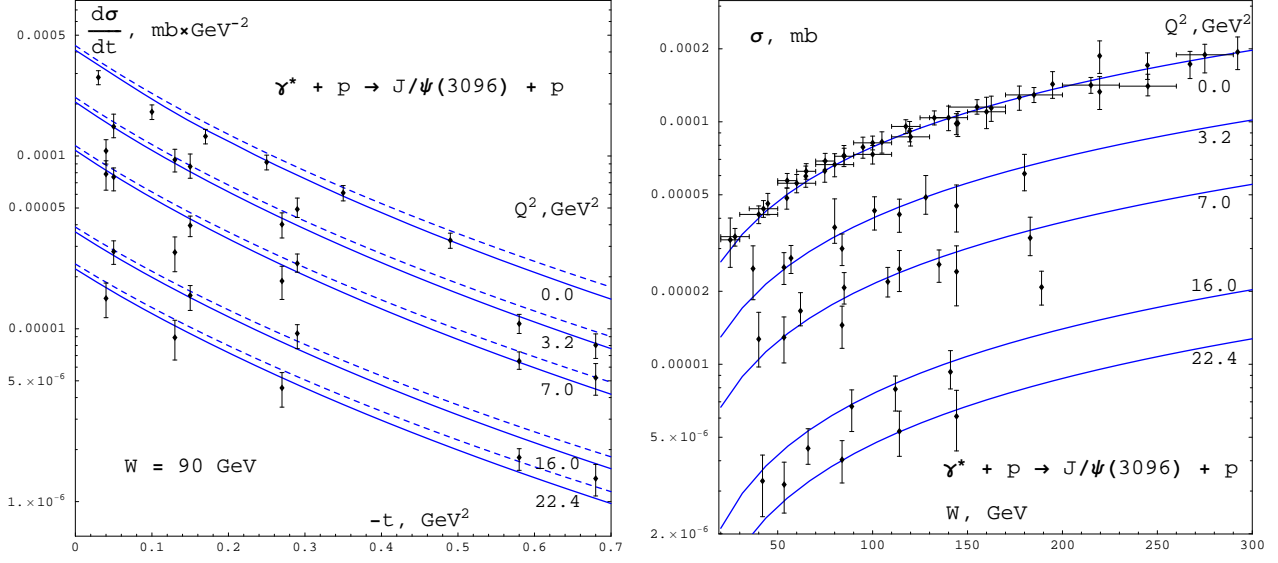


Figure 3: Differential and integrated cross-sections of the $J/\psi(3096)$ electroproduction at different values of the incoming photon virtuality. The dashed lines correspond to the Born amplitudes.

Q^2, GeV^2	0.0	3.2	7.0	16.0	22.4
$\Gamma_P^{(J/\psi^* J/\psi)}(Q^2)$	0.33	0.32	0.31	0.3	0.3
$\Gamma_f^{(J/\psi^* J/\psi)}(Q^2)$	0.19	0.17	0.15	0.11	0.1

Q^2, GeV^2	0.0	2.4	3.5	5.0	6.6	9.2	13.0	19.7
$\Gamma_P^{(\phi^* \phi)}(Q^2)$	0.6	1.5	1.6	1.6	1.6	1.6	1.6	1.4
$\Gamma_f^{(\phi^* \phi)}(Q^2)$	3.0	2.0	1.9	1.9	1.7	1.6	1.5	1.3

Q^2, GeV^2	0.0	2.5	3.5	5.0	6.0	6.6	8.0	11.7	13.5	17.4	19.7	33.0	35.6	41.0
$\Gamma_P^{(\rho^* \rho)}(Q^2)$	0.41	1.5	1.6	1.7	1.8	1.8	2.0	2.0	1.95	1.9	1.9	1.8	1.8	1.7
$\Gamma_f^{(\rho^* \rho)}(Q^2)$	4.2	3.7	3.6	3.3	3.1	3.0	2.75	2.3	2.2	1.8	1.7	1.1	1.0	0.9

Table 3: Reggeon form-factors of vector mesons at different values of the incoming photon virtuality.

Here we make an assumption that in the diffraction region one can neglect the t -dependence of reggeon form-factors of vector mesons (*i.e.* at low enough values of t the approximations $\Gamma_R^{(V^*V)}(t, Q^2) \approx \Gamma_R^{(V^*V)}(Q^2)$ and $\Gamma_R^{(VV)}(t) \approx \Gamma_R^{(VV)}$ are valid⁴). Such a behavior corresponds to the small effective radii of vector meson interaction with reggeons. According to the dimensional counting rules [16], the quantities $\Gamma_R^{(V^*V)}(Q^2)$ should exhibit slow (nonpower-like) Q^2 -behavior (power-like Q^2 -dependence was already singled out above). Taking into account this slow Q^2 -evolution we, also, assume that $\Gamma_R^{(VV)} \approx \Gamma_R^{(V^*V)}(0)$ (since we have no possibility to fix the on-shell form-factors separately from the elastic scattering of vector mesons on protons). Thus,

⁴The hypothesis about the weak t -dependence of vector meson reggeon form-factors (in comparison with corresponding form-factors of the proton) is confirmed *a posteriori* under describing the experimental data.

for estimation of the photoproduction ($Q^2 \approx 0$) amplitude we need to determine only two extra free parameters, $\Gamma_p^{(V^*V)}(0)$ and $\Gamma_f^{(V^*V)}(0)$.

Applying the model to the data on the photoproduction of $J/\psi(3096)$ on protons [17] yields $\chi^2 \approx 158$ over 114 points (see Fig. 2).

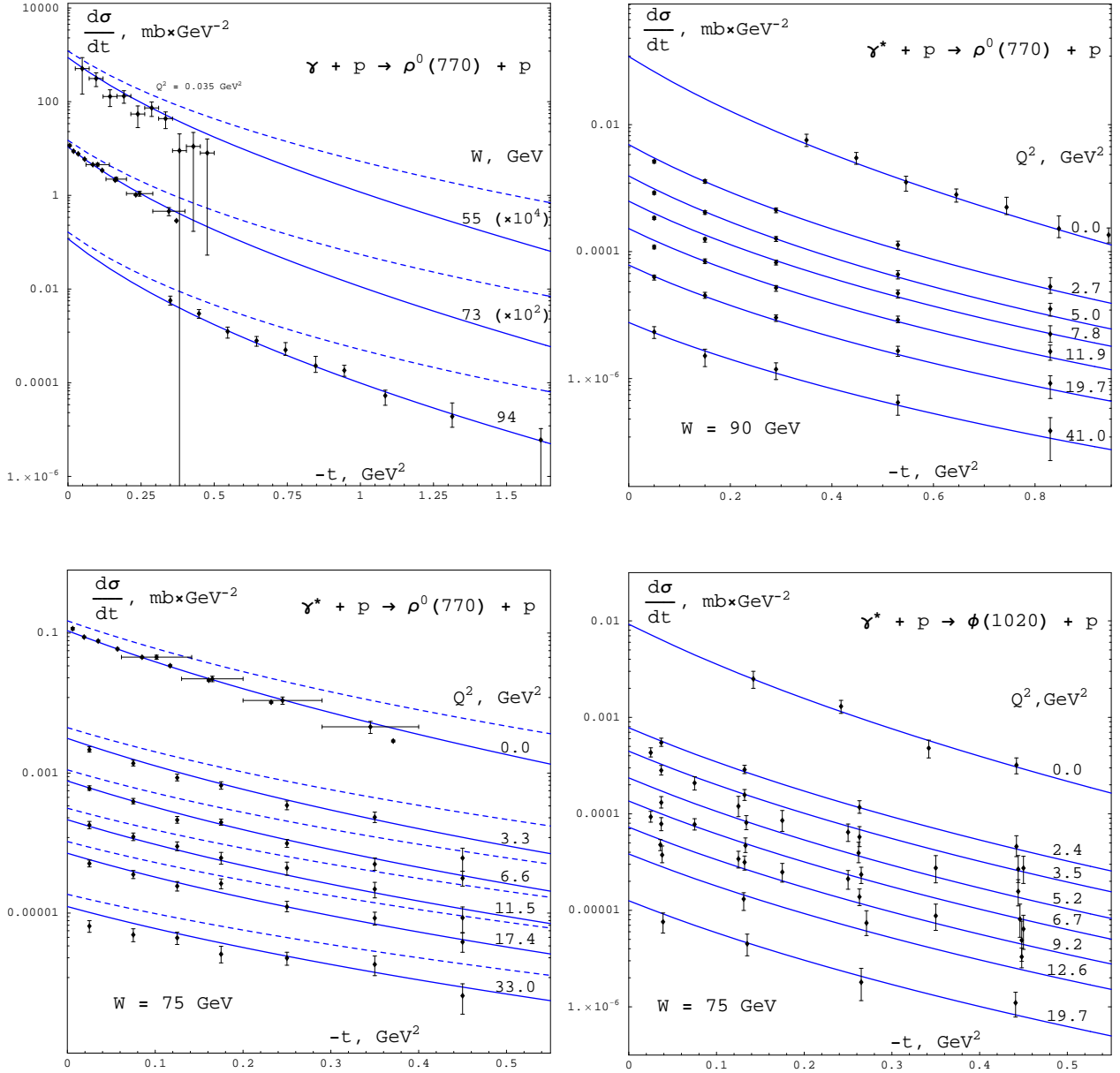


Figure 4: The t -behavior of the light vector meson electroproduction differential cross-sections at different virtualities of the incoming photon. The dashed lines correspond to the Born amplitudes.

For the electroproduction of $J/\psi(3096)$ [13, 17] and exclusive production of light vector mesons [18, 14] we obtain rather slow changing of reggeon form-factors with Q^2 growth (see Figs. 3,4,5⁵ and Tab. 3) in accordance with the dimensional counting rules [16].

For the photoproduction of $\psi(2s)$ [19] and $\Upsilon(1s)$ [20] we obtain $\Gamma_p^{(\psi^*\psi)}(0) \approx 0.25$, $\Gamma_f^{(\psi^*\psi)}(0) \approx 0.1$ and $\Gamma_p^{(\Upsilon^*\Upsilon)}(0) \approx 0.08$, $\Gamma_f^{(\Upsilon^*\Upsilon)}(0) \approx 0$ (see Fig. 6).

⁵Integrated cross-sections are obtained by integration of the differential ones over $0 < -t < 1 \text{ GeV}^2$ for heavy mesons and over $0 < -t < 0.6 \text{ GeV}^2$ for light mesons.

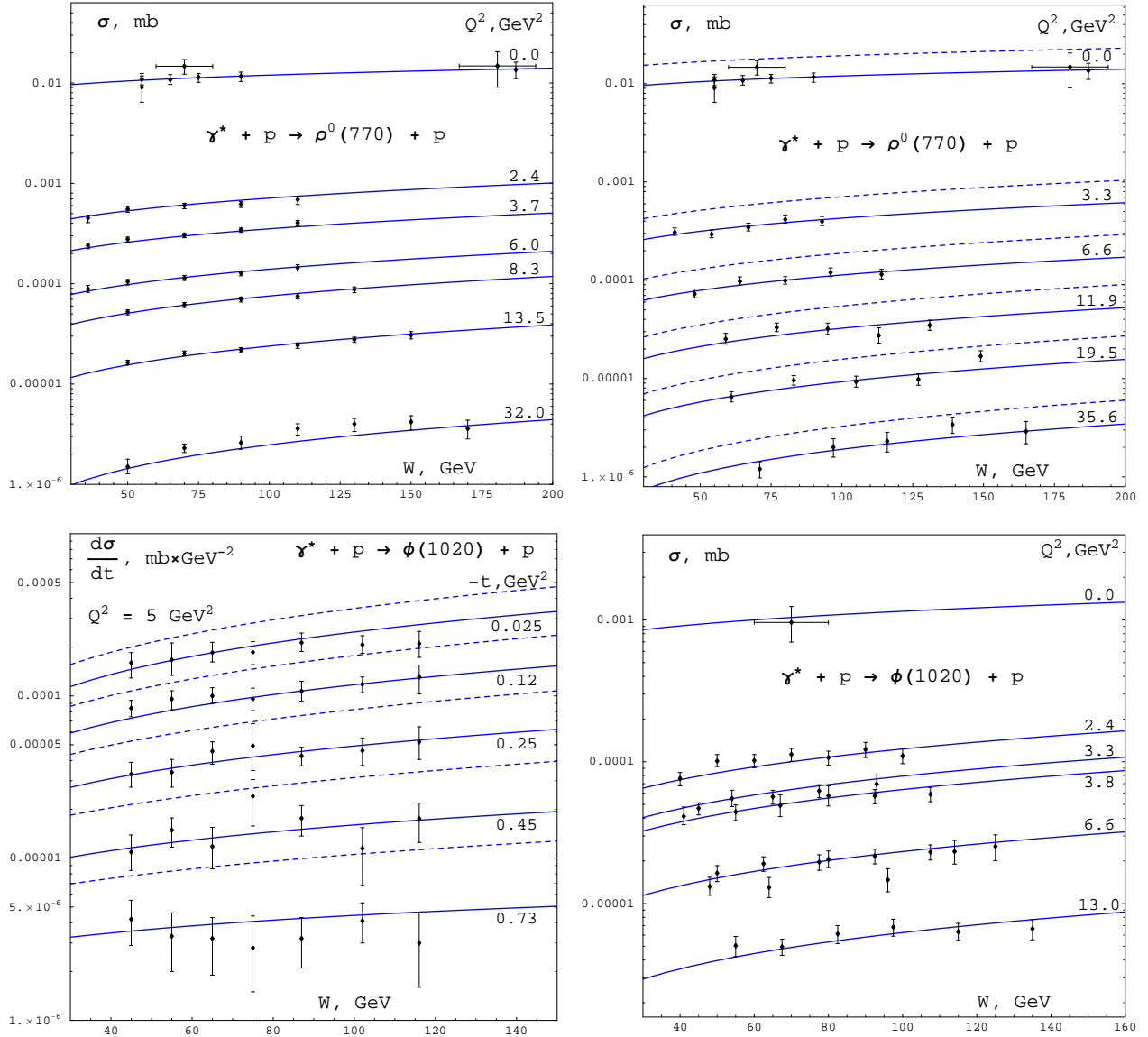


Figure 5: The W -behavior of the light vector meson electroproduction cross-sections at different virtualities of the incoming photon. The dashed lines correspond to the Born amplitudes.

High energy behavior of γ^*p total cross-sections

In this section we consider the W -behavior of the γ^*p total cross-sections which are closely related to the proton structure function $F_2(x, Q^2)$ [21]:

$$\sigma_{tot}^{\gamma^*p}(W^2, Q^2) = \frac{4\pi^2\alpha_e}{Q^4} \frac{Q^2 + 4m_p^2x^2}{1-x} F_2(x, Q^2) \quad \left(x = \frac{Q^2}{W^2 + Q^2 - m_p^2} \right). \quad (14)$$

Following the VDM the non-flip forward γ^*p scattering amplitude can be represented in the form:

$$T_{\gamma^*p \rightarrow \gamma^*p}^\lambda(W^2, t=0, Q^2) = \sum_{V', V} C_{V'}^\lambda(Q^2) T_{V'p \rightarrow Vp}(W^2, t=0, Q^2) C_V^\lambda(Q^2). \quad (15)$$

Owing to the optical theorem [1] the total cross-section is proportional to the imaginary part of the forward amplitude. From the reggeonic point of view one could single out (at high W) the pomeron pole contribution:

$$\sigma_{tot}^{\gamma^*p}(W^2, Q^2) = \Gamma_P^{(\gamma^*\gamma^*)}(Q^2) \Gamma_P^{(pp)}(0) W^{2(\alpha_P(0)-1)} + S + A, \quad (16)$$

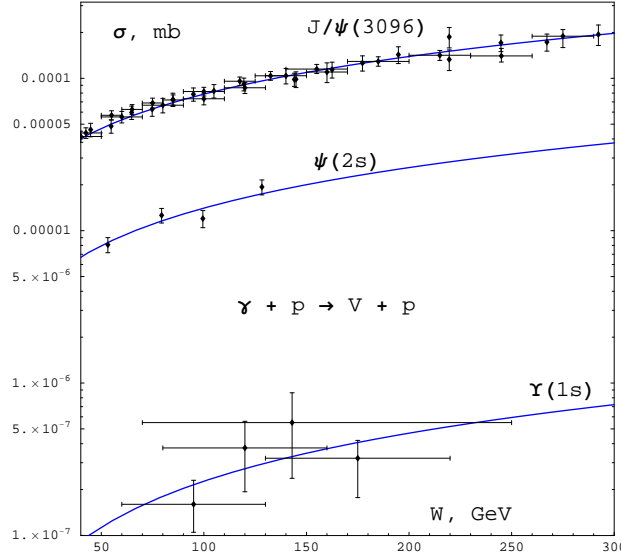


Figure 6: High energy behavior of the $\psi(2s)$ and $\Upsilon(1s)$ photoproduction cross-sections.

where S denotes the sum of other pole contributions, A denotes absorptive corrections, and $\Gamma_P^{(\gamma^*\gamma^*)}(Q^2)$ is the pomeron form-factor of virtual photon.

Immediately, a question emerges if there exists such a kinematical range at HERA energies where S and A are negligible and, hence, the simple pole approximation is valid ($W_0 \equiv 1$ GeV):

$$\sigma_{tot}^{\gamma^*p}(W^2, Q^2) \approx \beta(Q^2) \left(\frac{W}{W_0} \right)^{2\delta}, \quad (17)$$

where $\beta(Q^2) \equiv W_0^{-2} \Gamma_P^{(\gamma^*\gamma^*)}(Q^2) \Gamma_P^{(pp)}(0)$, $\delta \equiv \alpha_P(0) - 1$.

Such a range exists. At $50 \text{ GeV} < W < 300 \text{ GeV}$ and $25 \text{ GeV}^2 < Q^2 \ll W^2$ the W -behavior of the γ^*p total cross-sections can be well-described by (17) with $\delta \approx 0.31$ and values of $\beta(Q^2)$ from Tab. 4 (see Fig. 7).

$Q^2, \text{ GeV}^2$	12	15	20	25	35	45	60	90	120	150	200	250	300
$\beta(Q^2) \cdot 10^4, \text{ mb}$	5.0	4.1	3.1	2.45	1.7	1.32	0.97	0.61	0.44	0.33	0.23	0.18	0.145

$Q^2, \text{ GeV}^2$	350	400	500	650	800	1000	1200	1500	2000	3000	5000
$\beta(Q^2) \cdot 10^4, \text{ mb}$	0.117	0.1	0.075	0.054	0.042	0.031	0.026	0.018	0.012	0.007	0.0038

Table 4: The Q^2 -behavior of the pomeron residue for the γ^*p forward amplitude.

The slower rising of total cross-sections with the collision energy growth at lower Q^2 and W is due to the influence of secondary poles and absorptive corrections (the secondary pole terms rise cross-sections at lower energies and absorptive corrections drop them at higher energies). The fact that these contributions are negligibly small in the above-mentioned kinematical range could be explained by the following. Owing to the presence of factors $\frac{M_V^2}{M_V^2 + Q^2}$ in coefficients $C_V^\lambda(Q^2)$, at high photon virtualities the scattering amplitude (15) is dominated by fluctuations to heavy vector mesons. But for heavy mesons diffractive scattering the secondary pole terms and the absorptive corrections are suppressed by the pomeron pole term due to the much smaller f_2 -reggeon form-factors (in comparison with both the f_2 -reggeon form-factors of light mesons and the pomeron form-factors) and significantly lower values of the pomeron form-factors. This can be traced explicitly by the collation of the light meson ($\rho^0(770)$, $\phi(1020)$) versus the heavy meson ($J/\psi(3096)$, $\psi(2s)$, $\Upsilon(1s)$) exclusive production (see the previous section).

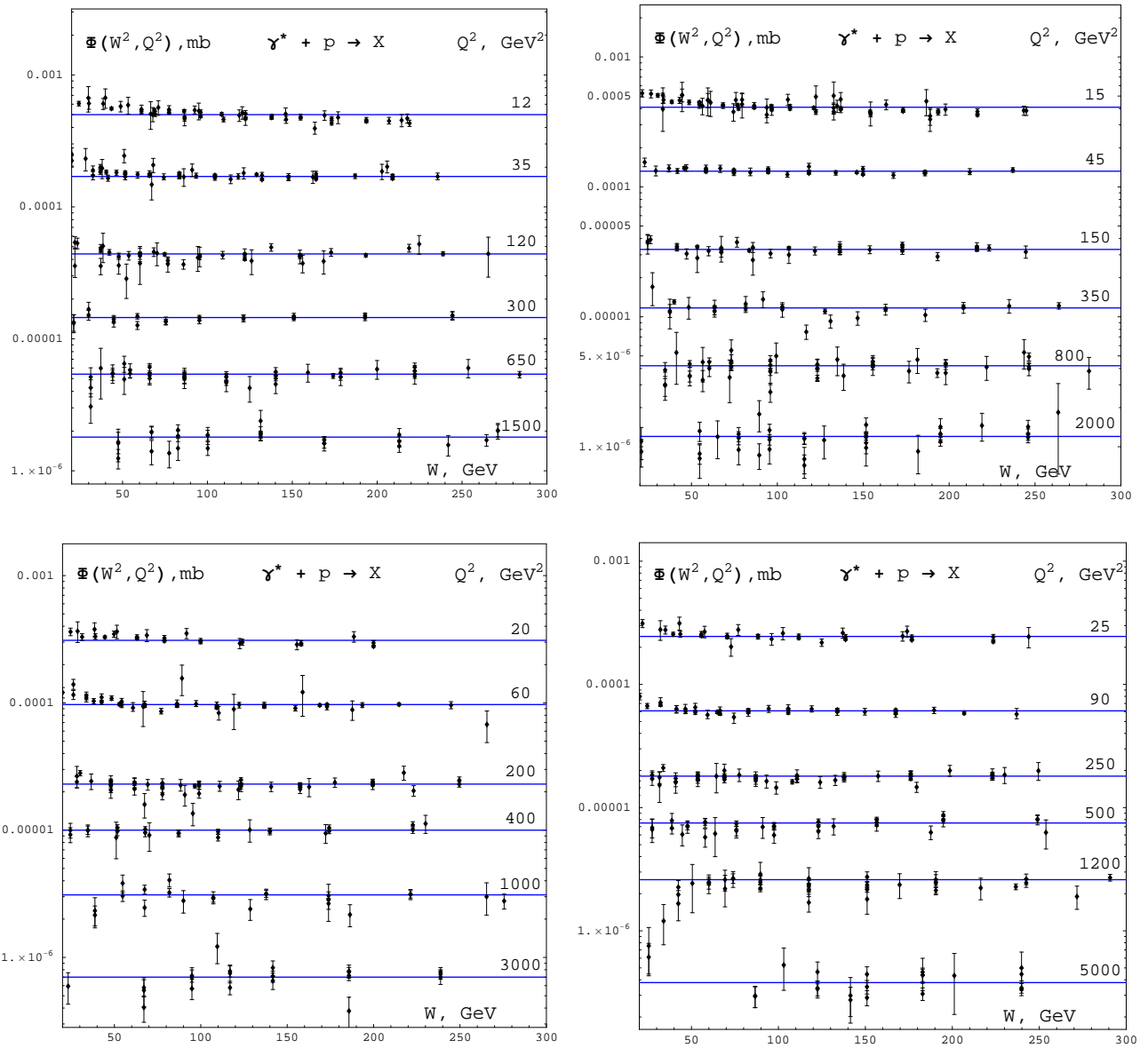


Figure 7: The W -behavior of the function $\Phi(W^2, Q^2) \equiv (W_0/W)^{0.62} \sigma_{tot}^{\gamma^* p}(W^2, Q^2)$ at different virtualities of the incoming photon.

After the extraction of δ from the data on $F_2(x, Q^2)$ and fixing the pomeron intercept we obtain (see the previous sections) a consistent phenomenological scheme for various exclusive diffractive reactions $2 \rightarrow 2$ at high energies.

Discussion

Now let us turn to the general discussion of the proposed model.

One of the advantages of Regge-eikonal approach is that it allows explicit taking into account absorptive corrections. Above there was demonstrated that absorptive corrections are not negligible for elastic nucleon-nucleon scattering and exclusive electroproduction of light vector mesons (see dashed lines in Figs. 1 – 5).

Regge trajectories in our model turn out essentially nonlinear in the diffraction region (see Fig. 1). At first sight such a picture may seem strange but such a nonlinearity is the only way for true Regge trajectories to combine the asymptotic behavior (3), (4) with the approximate

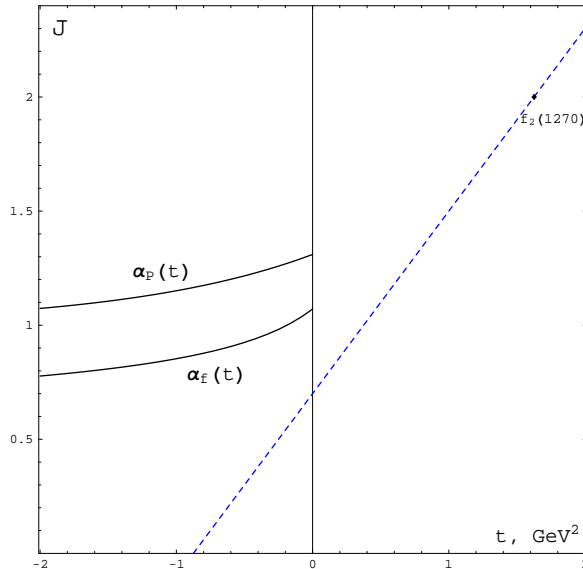


Figure 8: Test pomeron and f_2 -reggeon trajectories. The dashed line, $\alpha_f^{\text{lin}}(t) = 0.7 + 0.8t$, is the Chew-Frautschi plot for the f_2 -reggeon.

large-slope linearity in the resonance region⁶ (observed in the light meson spectroscopy data) and the monotony at negative values of the argument (see Fig. 8). This monotony itself seems quite natural. If $\text{Im } \alpha(t + i0) \geq 0$ increases slowly enough at $t \rightarrow +\infty$ (for example, not faster than $Ct \ln^{-1-\epsilon} t$, $\epsilon > 0$), the dispersion relations with not more than one subtraction take place, *i.e.*

$$\alpha(t) = \alpha_0 + \frac{t}{\pi} \int_{t_T}^{+\infty} \frac{\text{Im } \alpha(t' + i0)}{t'(t' - t)} dt'.$$

And if, in addition, we assume that $\text{Im } \alpha(t + i0) \geq 0$ at $t \geq t_T > 0$ (we would like to point out that these assumptions are strictly fulfilled in the theory of perturbations and the theory of potential scattering [1]) then

$$\frac{d^n \alpha(t)}{dt^n} > 0 \quad (t < t_T, \quad n = 1, 2, 3, \dots).$$

Also, using the nonlinear trajectories (instead of linear ones) allows to avoid emerging unphysical singularities in the real part of the signature factors at those points where Regge trajectories take on a value of negative integers.

The eikonal (5) could be called “minimal” since it contains contributions from only those reggeons which are essential for all considered diffractive reactions. For Regge trajectories and reggeon form-factors we chose parametrizations as simple as possible. Expressions (6), (7) were not derived from QCD or general principles. So, they should be considered not as analytic but only as purely quantitative (test) approximations to the true Regge trajectories and reggeon form-factors at negative values of the argument. They are not valid at $t > 0$. Moreover, we do not expect that exponential approximations (7) to reggeon form-factors are valid in the region of transfers essentially larger than 1 GeV (exponential form-factors correspond to Gaussian distribution of strongly interacting matter in nucleons). The imperfectness of these approximations and the neglect of secondary reggeons (ω , ρ , a , *etc.*) are the causes of systematic

⁶The resonance masses are determined from equations like $\alpha(M^2 - iM\Gamma) = J$ (here $\alpha(t)$ is some Regge trajectory and J is the resonance spin) which do not imply that $\text{Re } \alpha(M^2) = J$ and $\text{Im } \alpha(M^2) = 0$. The decay widths Γ are only few times less than the corresponding masses M : for example, for $f_2(1270)$ -meson $\frac{\Gamma_f}{M_f} \approx 0.15$. Consequently, Chew-Frautschi plots should be considered only as very rough approximations to the true Regge trajectories in the resonance region.

deviations of the model curves from the experimental data. Therefore, we had to consider only that kinematical range for the elastic nucleon-nucleon scattering where the contributions from secondary reggeons are small enough and where the non-exponential behavior of reggeon form-factors at high transfers is not essential.

For better description of the considered data sets on nucleon-nucleon scattering and for extension of the model to higher transfers one should use more complicated expressions for the reggeon form-factors. Also, for extension to lower energies and for description of the difference between pp and $\bar{p}p$ cross-sections it is necessary to include secondary reggeons (ω , ρ , a , *etc.*) and, possibly, odderon (which could have some influence on the behavior in the dip region). But in this job we deliberately restricted ourselves by the simplest eikonal and the simplest test parametrizations to make the main conclusions more transparent:

- For various diffractive processes there exist wide kinematical ranges where the cross-sections are dominated by only two reggeons, the pomeron and the f_2 -reggeon.
- Regge trajectories are universal in all diffractive reactions⁷ and essentially nonlinear (such a nonlinearity does not contradict to both the phenomenology and the QCD asymptotic relations).
- The pomeron trajectory has intercept ≈ 1.31 (which could be extracted from the data on the proton structure function $F_2(x, Q^2)$) and corresponds to the so-called “hard” pomeron frequently mentioned in literature. The f_2 -reggeon has intercept ≈ 1.07 and corresponds to the so-called “soft” pomeron.
- Different cross-section growth rates for various reactions (at high energies) are due to different relative contributions of the f_2 -reggeon to the eikonal.

Among other models for elastic diffractive scattering [22, 23, 24, 25, 26, 27] and exclusive electroproduction of vector mesons [28, 29, 30, 31, 32] the proposed phenomenological scheme stands out due to its salient simplicity and physical clearness and, also, due to the fact that it is the only model which allows to give a simultaneous qualitative description to both “soft” and “hard” exclusive diffractive processes. The range of validity of the model is wide enough to give a good ground for making well-reasoned predictions for elastic diffraction at LHC energies (see Fig. 1). The TOTEM measurements [33] on pp total cross-sections and angular distributions should discriminate among different models.

The obtained approximations to leading vacuum Regge trajectories and reggeon form-factors of the proton could be used under considering more complicated (than $2 \rightarrow 2$) diffractive reactions: single diffraction ($p + p \rightarrow p + X$ or $\bar{p} + p \rightarrow \bar{p} + X$), central exclusive diffractive production of the Higgs boson ($p + p \rightarrow p + H + p$), *etc.*

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Appendix. Derivation of the eikonal Regge approximation.

The eikonal representation of the scattering amplitude itself does not yield any progress in solving the problem since it is reduced to the replacement of the unknown function of two vari-

⁷This universality is closely related to the fact that Regge trajectories are analytic (*i.e.* unique) continuations of observable spectra of resonances.

ables, $T(s, t)$, to another one, $\delta(s, t)$, without any specification of the functional form of $\delta(s, t)$. The key assumption is that the eikonal is proportional (with high accuracy) to the effective relativistic (quasi-)potential of hadronic interaction. According to the Van Hove interpretation [34] of the relativistic (quasi-)potential as the “sum” over all single-particle exchanges in the t -channel, the eikonal can be represented in the form

$$\delta^{(f_1, f_2)}(s, t) = \sum_{j=0}^{\infty} \sum_{m_j} J_{\alpha_1 \dots \alpha_j}^{(f_1, j, m_j)}(p_1, \Delta) \frac{D_{(j, m_j)}^{\alpha_1 \dots \alpha_j, \beta_1 \dots \beta_j}(\Delta)}{m_j^2 - \Delta^2} J_{\beta_1 \dots \beta_j}^{(f_2, j, m_j)}(p_2, \Delta), \quad (\text{A.1})$$

(here $\frac{D_{(j, m_j)}^{\alpha_1 \dots \alpha_j, \beta_1 \dots \beta_j}}{m_j^2 - \Delta^2}$ is the propagator of the spin- j particle with mass m_j , $J_{\alpha_1 \dots \alpha_j}^{(f, j, m_j)}$ is the hadronic current (index f denotes the kind of scattering particle), Δ is the transferred 4-momentums, $t = \Delta^2$, p_1 and p_2 are 4-momentums of the incoming hadrons, $s = (p_1 + p_2)^2$, the symbol \sum_{m_j} denotes summing over all spin- j particles with different masses, which, in what further, will be transformed into summing over reggeons). We impose the following constraints on the general dependence of hadronic currents on Δ : symmetry with respect to all α_k , transversality with respect to Δ_{α_k} ($k = 1, \dots, j$), and tracelessness with respect to any pair of Lorentz indices. The first two conditions yield

$$J_{\alpha_1 \dots \alpha_j}^{(f, j, m_j)}(p, \Delta) = \sum_{k=0}^{\lfloor \frac{j}{2} \rfloor} \Gamma_k^{(f, j, m_j)}(p^2, \Delta^2, (p\Delta)) \sum G_{\alpha_{\mu_1} \alpha_{\mu_2}} \dots G_{\alpha_{\mu_{2k-1}} \alpha_{\mu_{2k}}} P_{\alpha_{\mu_{k+1}}} \dots P_{\alpha_{\mu_j}}, \quad (\text{A.2})$$

where $\Gamma_k^{(f, j, m_j)}(p^2, \Delta^2, (p\Delta))$ are some scalar functions, $P_\alpha \equiv \frac{p_\alpha - \frac{p\Delta}{\Delta^2} \Delta_\alpha}{\sqrt{p^2 - \frac{(p\Delta)^2}{\Delta^2}}}$, $G_{\alpha\beta} \equiv -g_{\alpha\beta} + \frac{\Delta_\alpha \Delta_\beta}{\Delta^2}$, and the inner sum is over all nonequivalent permutations of Lorentz indices (the total number of terms is $\frac{j!}{(2k)!!(j-2k)!}$). Taking into account that

$$g^{\alpha\beta} G_{\alpha\beta} = -3, \quad P_\alpha P^\alpha = 1, \quad g^{\alpha\beta} G_{\alpha\gamma} G_{\beta\delta} = -G_{\gamma\delta}, \quad G_{\alpha\beta} P^\beta = -P_\alpha, \quad (\text{A.3})$$

the tracelessness condition results in the recurrent relations

$$\Gamma_k^{(f, j, m_j)}(p^2, \Delta^2, (p\Delta)) = \frac{\Gamma_{k-1}^{(f, j, m_j)}(p^2, \Delta^2, (p\Delta))}{2(j-k) + 1}, \quad (\text{A.4})$$

and, consequently, yields

$$J_{\alpha_1 \dots \alpha_j}^{(f, j, m_j)}(p, \Delta) = \Gamma_0^{(f, j, m_j)}(p^2, \Delta^2, (p\Delta)) \times \sum_{k=0}^{\lfloor \frac{j}{2} \rfloor} \frac{(2(j-k) - 1)!!}{(2j-1)!!} \sum G_{\alpha_{\mu_1} \alpha_{\mu_2}} \dots G_{\alpha_{\mu_{2k-1}} \alpha_{\mu_{2k}}} P_{\alpha_{\mu_{k+1}}} \dots P_{\alpha_{\mu_j}}. \quad (\text{A.5})$$

Substituting (A.5) into (A.1) and taking into account the transversality and the tracelessness of hadronic currents, we come to the following expression for the eikonal:

$$\delta^{(f_1, f_2)}(s, t) = \sum_{j=0}^{\infty} \sum_{m_j} \frac{\gamma^{(f_1, f_2, j, m_j)}(\Delta^2)}{m_j^2 - \Delta^2} \frac{2^j (j!)^2}{(2j)!} P_j \left(\frac{p_1 p_2 - \frac{(p_1 \Delta)(p_2 \Delta)}{\Delta^2}}{\sqrt{(p_1^2 - \frac{(p_1 \Delta)^2}{\Delta^2})(p_2^2 - \frac{(p_2 \Delta)^2}{\Delta^2})}} \right), \quad (\text{A.6})$$

where $P_j(x)$ are Legendre polynomials of power j and

$$\gamma^{(f_1, f_2, j, m_j)}(\Delta^2) \equiv \Gamma_0^{(f_1, j, m_j)}(p_1^2, \Delta^2, (p_1 \Delta)) \Gamma_0^{(f_2, j, m_j)}(p_2^2, \Delta^2, (p_2 \Delta)). \quad (\text{A.7})$$

For elastic scattering we have

$$(p_1 - \Delta)^2 = p_1^2 = m_1^2, \quad (p_2 + \Delta)^2 = p_2^2 = m_2^2, \quad (\text{A.8})$$

and, so, using the kinematical relations

$$p_{1,2}\Delta = \pm \frac{\Delta^2}{2}, \quad p_1 p_2 = \frac{s - m_1^2 - m_2^2}{2}, \quad \Delta^2 \equiv t, \quad (\text{A.9})$$

we can simplify the expression for the eikonal (from now on, indices f_1 and f_2 , denoting the kinds of scattering particles, will be omitted):

$$\delta(s, t) = \sum_{j=0}^{\infty} \sum_{m_j} \frac{\gamma^{(j, m_j)}(t)}{m_j^2 - t} \frac{2^j (j!)^2}{(2j)!} P_j \left(\frac{s - m_1^2 - m_2^2 + \frac{t}{2}}{2\sqrt{(m_1^2 - \frac{t}{4})(m_2^2 - \frac{t}{4})}} \right). \quad (\text{A.10})$$

Here we divide the sum over j in the right-hand side of (A.10) into two sums over even and odd j :

$$\begin{aligned} \delta(s, t) = \delta^+(s, t) + \delta^-(s, t) = \sum_{j=0}^{\infty} \sum_{\eta=\pm 1} \sum_{m_{(\eta)j}} \frac{\eta + e^{-i\pi j}}{2} (-1)^j \frac{\gamma^{(\eta, j, m_{(\eta)j})}(t)}{m_{(\eta)j}^2 - t} \times \\ \times \frac{2^j (j!)^2}{(2j)!} P_j \left(\frac{s - m_1^2 - m_2^2 + \frac{t}{2}}{2\sqrt{(m_1^2 - \frac{t}{4})(m_2^2 - \frac{t}{4})}} \right). \end{aligned} \quad (\text{A.11})$$

Each of sequences $\gamma^{(\eta, j, m_{(\eta)j})}(t)$, $m_{(\eta)j}^2$ ($j = 0, 2, 4, \dots, 2n, \dots$ at $\eta = +1$ and $j = 1, 3, 4, \dots, 2n-1, \dots$ at $\eta = -1$) separately satisfies the conditions of the Carlson theorem [1, 35] which states that if for some analytic and regular at $\text{Re } x \geq 0$ function $f(x)$ the inequality $f(x) < e^{k|x|}$ at some $k < \pi$ is valid, then this function is determined unilocally by its values at integer x .

Now we make a key assumption about the possibility of unilocal (under the Carlson theorem) analytic continuation of (A.11) into the region of complex j (the Regge hypothesis [1]). It implies that $m_{(\eta)j}^2$ and $\gamma^{(\eta, j, m_{(\eta)j})}(t)$ in (A.11) are the values of analytic (holomorphic with respect to the complex variable j) functions for integer non-negative values of j . We denote these functions by $m_{\eta}^2(j)$ and $\gamma^{(\eta)}(t, j, m_{\eta}^2(j))$, respectively. Via the Sommerfeld–Watson transform [1, 36] we replace the sum over j in (A.11) by the integral over the contour C (on the complex plane of the variable j) encircling the real positive half-axis including the point $j = 0$ in such a way that the half-axis is on the right:

$$\begin{aligned} \delta(s, t) = \frac{1}{2i} \oint_C \frac{dj}{\sin(\pi j)} \sum_{\eta=\pm 1} \sum_{m_{\eta}} \left(-\frac{\eta + e^{-i\pi j}}{2} \right) \frac{\gamma^{(\eta)}(t, j, m_{\eta}^2(j))}{m_{\eta}^2(j) - t} \times \\ \times \frac{2^j \Gamma^2(j+1)}{\Gamma(2j+1)} P \left(j, \frac{s - m_1^2 - m_2^2 + \frac{t}{2}}{2\sqrt{(m_1^2 - \frac{t}{4})(m_2^2 - \frac{t}{4})}} \right) \end{aligned} \quad (\text{A.12})$$

(here $P(j, x)$ is the analytic continuation of Legendre polynomials $P_j(x)$ to the region of complex j and $\Gamma(j)$ is the Euler gamma-function). Since (according to our assumption) the unique sources of singularities of the integrand in the region $\text{Re } j > -\frac{1}{2}$ are the zeros of the functions $\sin(\pi j)$ and $m_{\eta}^2(j) - t$, then, by deforming the contour C and passing to the contour parallel to the imaginary axis, $\text{Re } j = -\frac{1}{2}$, we obtain

$$\delta(s, t) = \frac{1}{2i} \int_{-\frac{1}{2}-i\infty}^{-\frac{1}{2}+i\infty} \frac{dj}{\sin(\pi j)} \sum_{\eta=\pm 1} \sum_{m_{\eta}} \left(-\frac{\eta + e^{-i\pi j}}{2} \right) \frac{\gamma^{(\eta)}(t, j, m_{\eta}^2(j))}{m_{\eta}^2(j) - t} \times$$

$$\begin{aligned}
& \times \frac{2^j \Gamma^2(j+1)}{\Gamma(2j+1)} P \left(j, \frac{s - m_1^2 - m_2^2 + \frac{t}{2}}{2\sqrt{(m_1^2 - \frac{t}{4})(m_2^2 - \frac{t}{4})}} \right) + \\
& + \sum_{\eta=\pm 1} \sum_n \left(-\frac{\eta + e^{-i\pi\alpha_n^{(\eta)}(t)}}{\sin(\pi\alpha_n^{(\eta)}(t))} \right) \frac{d\alpha_n^{(\eta)}(t)}{dt} \frac{\pi\gamma^{(\eta)}(t, \alpha_n^{(\eta)}(t), t)}{2} \times \\
& \times \frac{2^{\alpha_n^{(\eta)}(t)} \Gamma^2(\alpha_n^{(\eta)}(t) + 1)}{\Gamma(2\alpha_n^{(\eta)}(t) + 1)} P \left(\alpha_n^{(\eta)}(t), \frac{s - m_1^2 - m_2^2 + \frac{t}{2}}{2\sqrt{(m_1^2 - \frac{t}{4})(m_2^2 - \frac{t}{4})}} \right),
\end{aligned} \tag{A.13}$$

where the functions $\alpha_n^{(\eta)}(t)$ are the roots of the equations $m_\eta^2(j) - t = 0$ and, thus, correspond to the poles of the eikonal in the region of complex j . These poles are called Regge poles, and the functions $\alpha_n^{(\eta)}(t)$ are called Regge trajectories (C -even at $\eta = +1$ and C -odd at $\eta = -1$).

For $s \gg m_1^2 + m_2^2 - \frac{t}{2}$ we can neglect the contribution of the background integral and the Legendre polynomials can be described by the leading terms of the expansion. Therefore, it is convenient to introduce new functions

$$\beta_n^{(\eta, s_0)}(t) \equiv \frac{d\alpha_n^{(\eta)}(t)}{dt} \frac{\pi\gamma^{(\eta)}(t, \alpha_n^{(\eta)}(t), t)}{2} \left(\frac{s_0}{2\sqrt{(m_1^2 - \frac{t}{4})(m_2^2 - \frac{t}{4})}} \right)^{\alpha_n^{(\eta)}(t)}, \tag{A.14}$$

where s_0 is any scale determined *a priori* (for example, $s_0 = 1 \text{ GeV}^2$). Note, that functions $\beta_n^{(\eta, s_0)}(t)$ depend on the chosen scale s_0 and can be factorized into two factors (see A.7) corresponding to the reggeon form-factors of the scattering particles. In the limit of high energies we obtain

$$\begin{aligned}
\delta(s, t) = & \sum_n \left(i + \text{tg} \frac{\pi(\alpha_n^+(t) - 1)}{2} \right) \Gamma_n^{(1)+}(t) \Gamma_n^{(2)+}(t) \left(\frac{s}{s_0} \right)^{\alpha_n^+(t)} \mp \\
& \mp \sum_n \left(i - \text{ctg} \frac{\pi(\alpha_n^-(t) - 1)}{2} \right) \Gamma_n^{(1)-}(t) \Gamma_n^{(2)-}(t) \left(\frac{s}{s_0} \right)^{\alpha_n^-(t)},
\end{aligned} \tag{A.15}$$

where the sign “−” (“+”) before C -odd contributions corresponds to the particle-particle (particle-antiparticle) scattering and $\Gamma_n^{(i)\pm}(t)$ are reggeon form-factors of the scattered particles.

The last formula⁸, together with the eikonal representation of the scattering amplitude (1), is the essence of the Regge-eikonal model [1].

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⁸It is valid, also, for those single-particle exchanges for which (A.8) is violated, *i.e.*, as for the inelastic scattering $2 \rightarrow 2$ so for reactions with off-shell particles.

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